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Essential obstacles to Helly circular-arc graphs. (English) Zbl 1504.05199

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The intersection graph of a set $\mathcal{A}$ of arcs on a circle is any graph isomorphic to the one having $\mathcal{A}$ as the vertex set and with an edge between two arcs whenever they intersect. A circular-arc graph G is an intersection graph of some set $\mathcal{A}$ of arcs on a circle; such a set $\mathcal{A}$ is also called a circular-arc model of the graph G. A Helly circular-arc graph is a circular-arc graph that has a circular-arc model satisfying the Helly property: every nonempty subfamily that is pairwise intersecting has a nonempty total intersection.

B. L. Joeris et al. [Algorithmica 59, No. 2, 215–239 (2011; Zbl 1209.68376)] characterized Helly circular-arc graphs to be precisely those circular-arc graphs that do not contain any induced subgraph belonging to the family of graphs called obstacles. However, the family of obstacles is neither minimal – since there exist obstacles that are induced subgraphs of other obstacles – nor contained in the family of circular-arc graphs. This naturally raises the question of characterizing the minimal circular-arc obstacles. *F. Bonomo* et al. [Discrete Math. Theor. Comput. Sci. 16, No. 3, 1–22 (2014; Zbl 1294.05079)] produced a partial list of these graphs. In this paper, the author introduces a refinement of the notion of obstacle, called essential obstacle, and proves that the minimal forbidden induced circular-arc subgraphs for the class of Helly circular-arc graphs are precisely the essential obstacles. Furthermore, it is shown that, given any obstacle, one can find in linear time a minimal forbidden induced subgraph for the class of Helly circular-arc graphs contained in it as an induced subgraph. In particular, using the linear time certifying algorithm of *B. L. Joeris* et al. [Algorithmica 59, No. 2, 215–239 (2011; Zbl 1209.68376)], one can also obtain in linear time a minimal negative certificate for input as any circular-arc graph that is not a Helly circular-arc graph.

It is an open problem to find a forbidden induced subgraph characterization for the class of Helly circular-arc graphs, i.e. not restricted only to circular-arc graphs. As a partial answer, the author obtains the minimal forbidden induced subgraph characterization for the class of Helly circular-arc graphs restricted to graphs containing no induced claw and no induced 5-wheel. Furthermore, it is shown that one can find in linear time an induced claw, an induced 5-wheel, or an induced minimal forbidden induced subgraph for the class of Helly circular-arc graphs in any given graph that is not a Helly circular-arc graph.

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MSC:

05C62 Graph representations (geometric and intersection representations, etc.)
05C75 Structural characterization of families of graphs
05C85 Graph algorithms (graph-theoretic aspects)

Keywords:

circular-arc graphs; forbidden subgraphs; Helly property; claw-free graphs

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